Digital Image Processing and Pattern Recognition

E1528

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Lecture 5

Fundamentals of Spatial Filtering

(Correlation & Convolution & Padding)

INSTRUCTOR

DR / AYMAN SOLIMAN

➢ Contents

Introduction

- Spatial filtering is used in a broad spectrum of image processing applications, so a solid understanding of filtering principles is important.
- The name filter is borrowed from frequency domain processing where "filtering" refers to passing, modifying, or rejecting specified frequency components of an image.
- For example, a filter that passes low frequencies is called a lowpass filter. The net effect produced by a lowpass filter is to smooth an image by blurring it. We can accomplish similar smoothing directly on the image itself by using spatial filters.

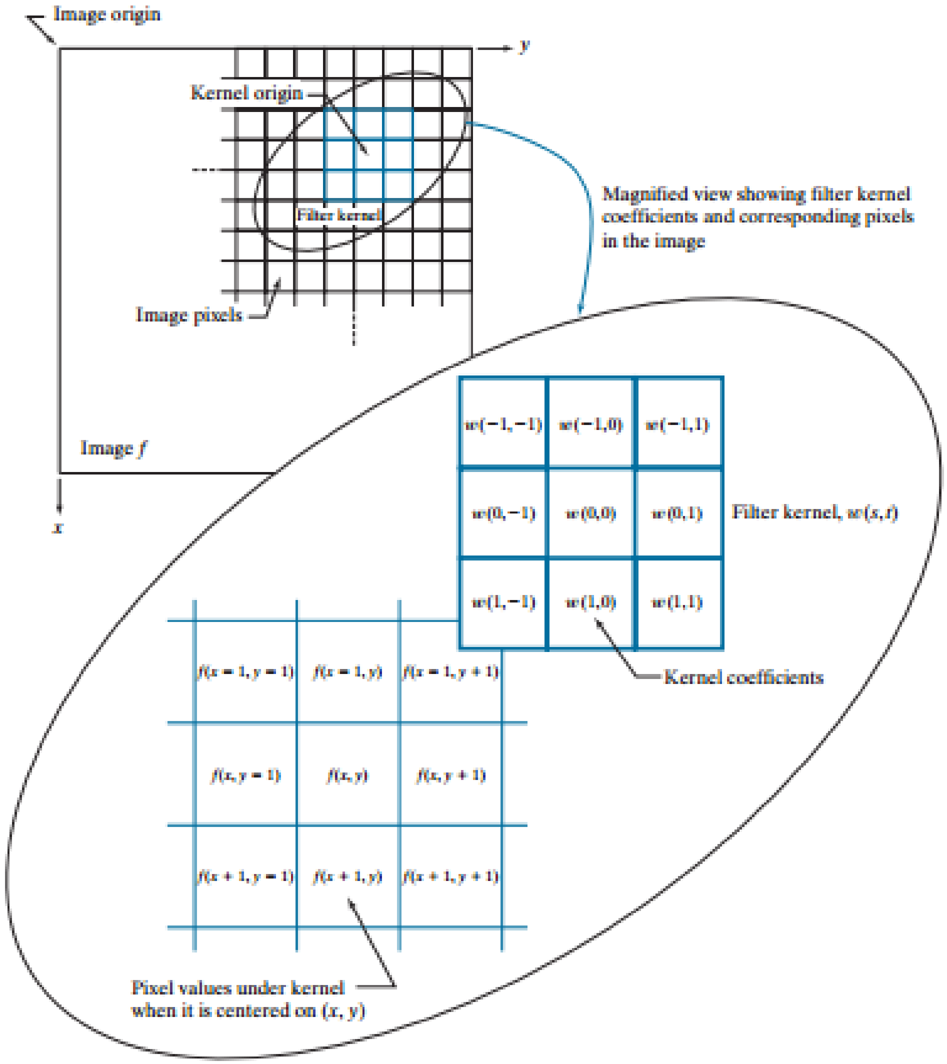
Spatial Filters

- Spatial filtering modifies an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors.
- If the operation performed on the image pixels is linear, then the filter is called a linear spatial filter. Otherwise, the filter is a nonlinear spatial filter.
- We will focus attention first on linear filters and then introduce some basic nonlinear filters.

➢ The Mechanics of Linear Spatial Filtering

- ➢ A linear spatial filter performs a sum-of-products operation between an image f and a filter kernel, w.
- ➢ The kernel is an array whose size defines the neighborhood of operation, and whose coefficients determine the nature of the filter.
- ➢ Other terms used to refer to a spatial filter kernel are mask, template, and window. We use the term filter kernel or simply kernel.

➢ The Mechanics of Linear Spatial Filtering



➢ The Mechanics of Linear Spatial Filtering

- ➢ Last figure illustrates the mechanics of linear spatial filtering using a 3 × 3 kernel. At any point (x,y) in the image, the response, g(x,y), of the filter is the sum of products of the kernel coefficients and the image pixels encompassed by the kernel:

$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \ldots + w(0,0)f(x,y) + \ldots + w(1,1)f(x+1,y+1)$$

- ➢ As coordinates x and y are varied, the center of the kernel moves from pixel to pixel, generating the filtered image, g, in the process.

The Mechanics of Linear Spatial Filtering

- Observe that the center coefficient of the kernel, w(0,0), aligns with the pixel at location (x,y). For a kernel of size m × n , we assume that m = 2a+1 and n = 2b+1, where a and b are nonnegative integers.

- This means that our focus is on kernels of odd size in both coordinate directions.

- In general, linear spatial filtering of an image of size M × N with a kernel of size m × n is given by the expression

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \qquad (1)$$

Spatial Correlation and Convolution

- Spatial correlation is illustrated graphically in last Figure, and it is described mathematically by equation (1).
- Correlation consists of moving the center of a kernel over an image and computing the sum of products at each location.
- The mechanics of spatial convolution are the same, except that the correlation kernel is rotated by 180°. Thus, when the values of a kernel are symmetric about its center, correlation and convolution yield the same result. The reason for rotating the kernel will become clear in the following discussion. The best way to explain the differences between the two concepts is by example.

➢ Spatial Correlation and Convolution (cont.)

➢ We begin with a 1-D illustration, in which case equation(1) becomes

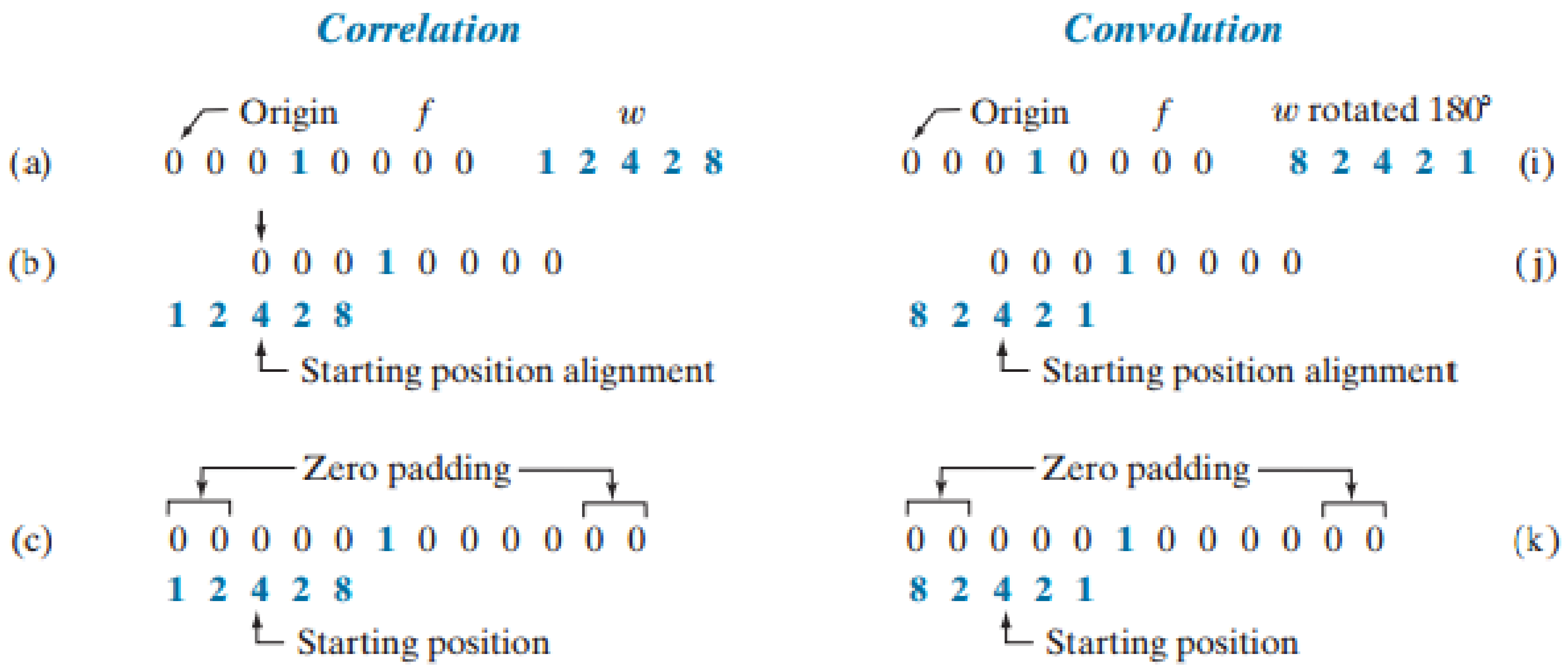
$$g(x) = \sum_{s=-a}^{a} w(s) f(x+s) \qquad (2)$$

Next figure (a) shows a 1-D function, f, and a kernel, w. The kernel is of size 1 × 5 , so a = 2 and b = 0 in this case. Figure (b) shows the starting position used to perform correlation, in which w is positioned so that its center coefficient is coincident with the origin of f.

Spatial Correlation and Convolution (cont.)

- The first thing we notice is that part of w lies outside f, so the summation is undefined in that area.
- A solution to this problem is to pad function f with enough 0's on either side. In general, if the kernel is of size $1 \times m$, we need $(m-1)/2$ zeros on either side of f in order to handle the beginning and ending configurations of w with respect to f.
- Figure(c) shows a properly padded function. In this starting configuration, all coefficients of the kernel overlap valid values.

➢ Spatial Correlation and Convolution (cont.)



➢ Spatial Correlation and Convolution (cont.)

(d) 0 0 0 0 0 1 0 0 0 0 0 0

1 2 4 2 8

Position after 1 shift

(e) 0 0 0 0 0 1 0 0 0 0 0 0

1 2 4 2 8

Position after 3 shifts

(f) 0 0 0 0 0 1 0 0 0 0 0 0

1 2 4 2 8

Final position

Correlation result

(g) 0 8 2 4 2 1 0 0

Extended (full) correlation result

(h) 0 0 0 8 2 4 2 1 0 0 0 0

0 0 0 0 0 1 0 0 0 0 0 0 (l)

8 2 4 2 1

Position after 1 shift

0 0 0 0 0 1 0 0 0 0 0 0 (m)

8 2 4 2 1

Position after 3 shifts

0 0 0 0 0 1 0 0 0 0 0 0 (n)

8 2 4 2 1

Final position

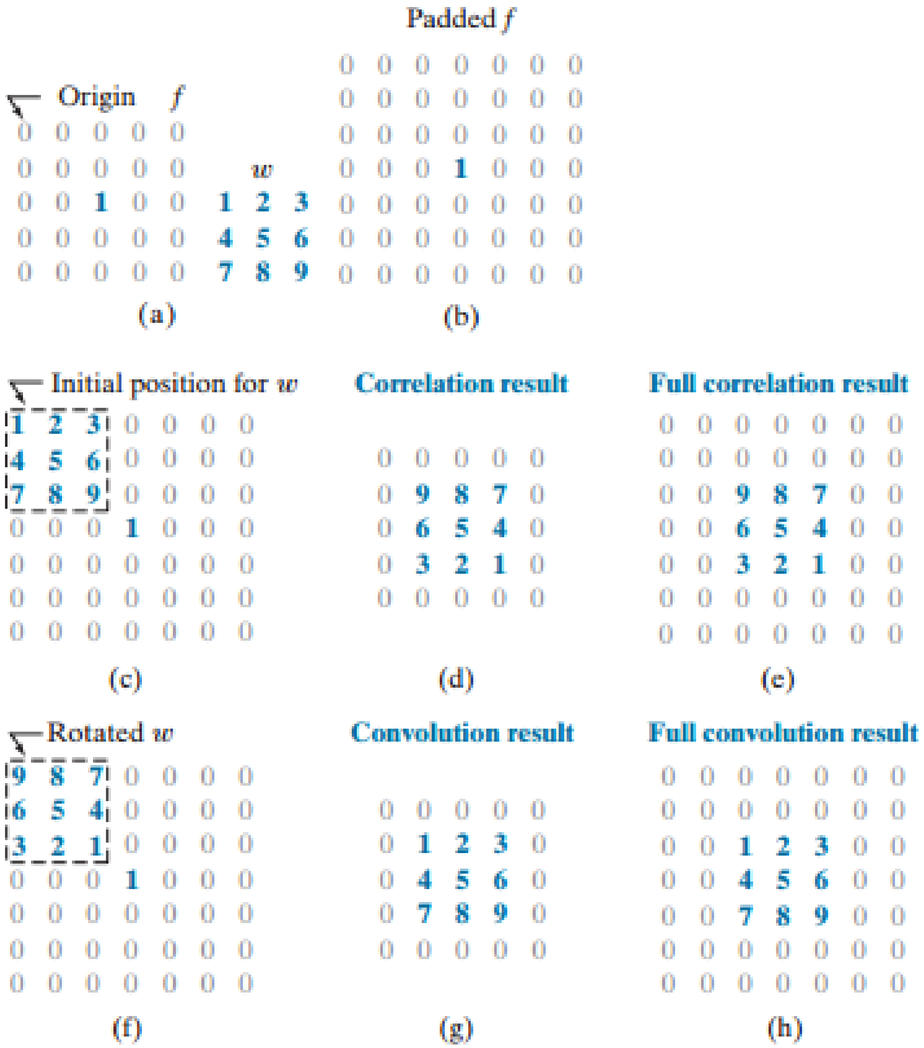
Convolution result

0 1 2 4 2 8 0 0 (o)

Extended (full) convolution result

0 0 0 1 2 4 2 8 0 0 0 0 (p)

➢ Spatial Correlation and Convolution 2-D



Properties

- Some fundamental properties of convolution and correlation.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

- Sometimes an image is filtered (i.e., convolved) sequentially, in stages say Q stage. Because of the commutative property of convolution, this multistage filtering can be done in a single filtering operation, as following.

- Note that We cannot do the same reduction for correlation because it is not commutative.

Padding

- What is Padding?
- Types of padding.
- Comparison

What is Padding?

- Padding extends the boundaries of an image to avoid undefined operations when parts of a kernel (mask) lie outside the border of the image during filtering.

- In general, if the kernel is of size m*n, we need (m-1)/2 rows at the top and bottom and (n-1)/2 columns at the right and left and the values these new cells will take depends on the padding technique.

Types of padding

- Zero padding
- Mirror (or symmetric) padding
- Replicate padding

zero padding

- In signal processing, zero padding refers to the practice of adding zeroes to a time-domain signal.

- Zero-padding is often done before performing a fast Fourier transform on the time-domain signal.

$$\mathbf{X} = \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix},$$

$$\mathrm{Pad}(1, \mathbf{X}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & a & b & c & 0 \\ 0 & d & f & g & 0 \\ 0 & h & j & k & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

➢ Mirror padding

- ➢ Values outside the boundary of the image are obtained by mirror-reflecting the image across its border.

13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8
3	2	1	2	3	4	5	4	3
8	7	6	7	8	9	10	9	8
13	12	11	12	13	14	15	14	13
18	17	16	17	18	19	20	19	18
13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8

➢ Replicate padding

- ➢ Values outside the boundary are set equal to the nearest image border value. It is useful when the areas near the border of the image are constant.

1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
6	6	6	7	8	9	10	10	10
11	11	11	12	13	14	15	15	15
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20

➢ Comparison

- Zero Padding: When zero (black) padding is used, the net result of smoothing at or near the border is a dark gray border that arises from including black pixels in the averaging process.

- Using the 11*11 kernel resulted in a more prominent dark border. The result with the 21*21 the kernel shows significant blurring of all components of the image, including a darker boarder.

- The two other approaches to padding tend to solve the dark-border problem that results from zero padding.

Comparison

- Mirror Padding: mirror padding does duplicate the details of the image on the edges which makes it more applicable when the areas near the border contain image details.

- Replicate Padding: It duplicate the only pixel on the edge which makes it useful when the areas near the border of the image are constant.


Thank
you